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NEW VIBRATIONS WITHIN A CONICAL HORN

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In the case of simple harmonic motion, the time factor being $e^{i\sigma t}$, the velocity potential Φ of the vibrations must satisfy the equation

$$(\Delta^2 + k^2)\,\Phi = 0$$

Where $k = \sigma/c$ in which c is the velocity of sound. As a solution we may take

$$\Phi = (kr)^{-1/2} J_{n+1/2}(kr)\, P_m\,(\cos\Theta)$$

where J and P denote Bessel and Legendre functions respectively and where r and Θ are polar coördinates. Suppose that the cone $\Theta = \Theta_0$ is a rigid boundary. Since this surface must be orthogonal to the equipotential surfaces, we must have

$$\frac{\partial\Phi}{\partial\Theta}$$

This is satisfied identically for $m = 0$, since $P_0 = 1$ and hence $\Phi = kr^{-1/2} J_{1/2}(kr)$ is a suitable solution. In this case the equipotential surfaces are spheres. Other values of m may be found as follows. The function $P'_m(\cos\Theta_0)$ may be developed in a power series in $1/m$. Equating this series to zero and reverting the series we obtain

$$m = \frac{y}{2\xi} - \frac{1}{2} + \left[\frac{-y}{3} + \frac{1}{y}\right]\xi/4 + \left[-\frac{17}{90}y + \frac{13}{15y} - \frac{1}{2y^3}\right]\frac{\xi^3}{8} + \left[\frac{299}{945}y^2 - \frac{1}{3} + \frac{1}{y^2} - \frac{1}{y^4}\right]\frac{\xi^4}{8}$$

where $\xi = \sin\frac{\Theta_0}{2}$ and y is a root of J_1.

If we assume that the inertia of the air outside the horn is negligible, there will be no excess pressure at the opening and therefore Φ will be zero there. Hence we must have:

$$J_{m+1/2}(k\,l) = 0$$

where l is the length of the horn. Using an expansion due to J. W. Nicholson (Phil. Mag. Vol. 14, p. 697, 1907), a sequence of values of k may be obtained from the above equation and, corresponding to each, a value of λ and n, the wave length and frequency. For $m = 0$, we have:

$$(k\,l)^{-1/2} J_{1/2}(k\,l) = \frac{\sqrt{2}}{\pi}\;\frac{\sin k\,l}{k\,l} = 0$$

and therefore $k = p\pi/l$ where p is a positive integer. For $p = 1$, we find $n = \frac{c}{2l} = n_o$, the frequency of the fundamental vibration.

Let the vibration corresponding to y, the smallest root of J_1 be designated as type I and that corresponding to y the next larger root of J_1 as type II. Let λ_s and m_s correspond to k_s where k_s is the s^{th} value of k in the sequence of values arranged in increasing order of magnitude. Let d be the diameter of the opening of the horn. As explained above, the following table may be obtained:

Vibration of Type I

Θ_0	n_1/n_0	n_2/n_0	n_3/n_0	λ_1/d	λ_2/d	λ_3/d
2°	37.85	40.11	41.99	.7572	.7143	.6823
4°	19.80	21.66	23.25	.7238	.6617	.6166
6°	13.71	15.38	16.83	.6977	.6221	.5685
8°	10.63	12.18	13.55	.6760	.5895	.5302
10°	8.763	10.24	11.54	.6570	.5624	.4987
15°	6.242	7.595	8.811	.6189	.5087	.4386
20°	4.96	6.24	7.40	589	.468	.395
25°	4.18	5.41	6.54	.566	.437	.362
30°	3.65	4.70	5.96	.548	.425	.336

Vibration of Type II

Θ_0	n_1/n_0	n_2/n_0	n_3/n_0	λ_1/d	λ_2/d	λ_3/d
2°	67.51	70.19	72.48	.4244	.4082	.3954
4°	34.79	36.99	38.87	.4120	.3876	.3688
6°	23.80	25.77	27.44	.4019	.3712	.3489
8°	18.26	20.08	21.63	.3934	.3578	.3322
10°	14.92	16.63	18.10	.3860	.3463	.3181
15°	10.41	11.96	13.31	.3711	.3230	.2902
20°	8.13	9.58	10.82	.359	.305	.270
25°	6.75	8.13	9.36	.351	.291	.253
30°	5.82	7.15	8.35	.344	.280	.239

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